19. LHS

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= 2\begin{vmatrix} a & -b & -c \\ p & -q & -r \\ x & -y & -z \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

Hence Proved

20. $x = a \sin 2t (1 + \cos 2t)$

$\frac{dx}{dt} = 2a \cos 2t (1 + \cos 2t) - 2a \sin^2 2t$

$= 2a \cos 2t + 2a \cos^2 2t - 2a \sin^2 t$

$= 2a \cos 2t + 2a \cos 4t$

$y = b \cos 2t (1 - \cos 2t)$

$\frac{dy}{dt} = - 2b \sin 2t (1 - \cos 2t) + b \cos 2t (2 \sin 2t)$

$= - 2b \sin 2t + 2b \sin 2t \cos 2t + 2b \sin 2t \cos 2t$

$= - 2b \sin 2t + 2b \sin 4t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \dfrac{2b\sin 4t - 2b\sin 2t}{2a\cos 2t + 2a\cos 4t}$

$= \dfrac{b}{a}\dfrac{(\sin 4t - 2b\sin 2t)}{\cos 2t + 2a\cos 4t}$

$\left(\dfrac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \dfrac{b}{a}\dfrac{(\sin\pi - \sin\frac{\pi}{2})}{\cos\frac{\pi}{2} + \cos\pi}$

$= \dfrac{b}{a}\dfrac{(-1)}{(-1)} = \dfrac{b}{a}$ hence proved

21. $x(1 + y^2)\,dx - y(1 + x^2)\,dy = 0$
$y(1 + x^2)\,dy = x(1 + y^2)\,dx$

$\dfrac{ydy}{1+y^2} = \dfrac{x}{1+x^2}\,dx$

Integrating both sides

$\int y\dfrac{dy}{l+y^2} = \int\dfrac{x}{l+x^2}\,dx$

$\dfrac{1}{2}\int\dfrac{2y}{1+y^2}\,dy = \dfrac{1}{2}\int\dfrac{2x}{1+x^2}\,dx$

$\dfrac{1}{2}\log|1 + y^2| = \dfrac{1}{2}\log|1 + x^2| + \log C$

$\log|1 + y^2| = \log|1 + x^2| + 2\log C$
$\log|1 + y^2| = \log|C^2(1 + x^2)|$
when $y = 1$, $x = 0$
$2 = C^2$
$\therefore 1 + y^2 = 2(1 + x^2)$
$y^2 = 2(1 + x^2)$
$y^2 = 2x^2 + 1$
$y = \sqrt{2x^2+1}$

22. Equation of staight line passing through (2, 1, 3) is

$\dfrac{x-2}{a} = \dfrac{y-1}{b} = \dfrac{z-3}{c}$

It is perpendicular to the lines
$\therefore a + 2b + 3c = 0$
$-3a + 2b + 5c = 0$

$\dfrac{a}{10-6} = \dfrac{-b}{5+9} = \dfrac{c}{2+6}$

$\dfrac{a}{4} = \dfrac{b}{-14} = \dfrac{c}{8}$

$\dfrac{a}{2} = \dfrac{b}{-7} = \dfrac{c}{4}$

$\therefore$ Equation of straight line is

$\dfrac{x-2}{2} = \dfrac{y-1}{-7} = \dfrac{z-3}{4}$

cartesian equation
Vector equation

$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$

28. $\sqrt{\tan x} + \sqrt{\cot x}\ dx$

$= \int\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$

$= \sqrt{2}\int\frac{\sin x + \cos x}{\sqrt{2\sin x\cos x}}dx$

$= \sqrt{2}\int\frac{\sin x + \cos x}{\sqrt{1-[1-2\sin x\cos x]}}dx$

$= \sqrt{2}\int\frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}dx$

Let sinx – cosx = t

(cosx + sinx)dx = dt

$= \sqrt{2}\int\frac{dt}{\sqrt{1-t^2}}$

$= \sqrt{2}\sin^{-1}(t) + C$

$= \sqrt{2}\sin^{-1}(\sin x - \cos x) + C$

29. $R^2 = r^2 + \frac{h^2}{4} \Rightarrow r^2 = R^2 - \frac{h^2}{4}$

$V = \pi r^2 h$

$V = \pi(R^2 - \frac{h^2}{4})\ h$

$V = \pi R^2 h - \pi\frac{h^3}{4}$

Diff. w.r.t h

$\frac{dv}{dh} = \pi R^2 - \pi\frac{3h^2}{4}$

For the critical point

$\frac{dv}{dh} = 0$

$\pi R^2 - \pi\frac{3h^2}{4} = 0$

$\pi\frac{3h^2}{4} = \pi R^2$

$h^2 = \frac{4R^2}{3}$

$h = \frac{2R}{\sqrt{3}}$

$\frac{dv}{dh} = \pi R^2 - \pi\frac{3h^2}{4}$

Again diff. w.r.t h

$\frac{d^2v}{dh^2} = -\frac{6\pi h}{4} < 0$

Volume is maximum when height of cylinder is

$\frac{2R}{\sqrt{3}}$